Homework Problems
Physics 417/517
Due March 16, 2021

1. What is the phase advance per pass in a $\nu = 1$ microtron with (stable) synchronous phase $\phi_s = +10^\circ$ after the crest phase?

2. Show explicitly that the matrix representation

$$M_{s',s} = \begin{pmatrix} \sqrt{\dfrac{\beta(s')}{\beta(s)}}\left(\cos\Delta\mu_{s',s} + \alpha(s)\sin\Delta\mu_{s',s}\right) & \sqrt{\beta(s')\beta(s)}\sin\Delta\mu_{s',s} \\ -\dfrac{1}{\sqrt{\beta(s')\beta(s)}}\begin{bmatrix}\left(1+\alpha(s')\alpha(s)\right)\sin\Delta\mu_{s',s} \\ +\left(\alpha(s')-\alpha(s)\right)\cos\Delta\mu_{s',s}\end{bmatrix} & \sqrt{\dfrac{\beta(s)}{\beta(s')}}\left(\cos\Delta\mu_{s',s} - \alpha(s')\sin\Delta\mu_{s',s}\right) \end{pmatrix}.$$

satisfies the composition formula $M_{s'',s} = M_{s'',s'}M_{s',s}$. Also, show from this representation that

$$\tan\Delta\mu_{s',s} = \frac{\left(M_{s',s}\right)_{12}}{\beta(s)\left(M_{s',s}\right)_{11} - \alpha(s)\left(M_{s',s}\right)_{12}}.$$

(Hint: The easy way is to use Floquet coordinates. The hard way is to brute force the matrix multiplies. You will need $\cos(A+B) = \cos A\cos B - \sin A\sin B$ and $\sin(A+B) = \sin A\cos B + \cos A\sin B$).

3. As was done in class for the focusing direction, derive the fundamental solution for the dispersion for a region of length L with constant defocusing ($k < 0$)

$$D_{p,0}(s) = \frac{1}{(-k)\rho}\left(\cosh\sqrt{-k}s - 1\right)$$
$$D'_{p,0}(s) = \frac{1}{\sqrt{-k}\rho}\sinh\sqrt{-k}s$$
.

Show the 3×3 dispersion tracking matrix is

$$\begin{pmatrix} \cosh\sqrt{-k}L & \sinh\sqrt{-k}L/\sqrt{-k} & \dfrac{1}{(-k)\rho}\left(\cosh\sqrt{-k}L - 1\right) \\ \sqrt{-k}\sinh\sqrt{-k}L & \cosh\sqrt{-k}L & \dfrac{1}{\sqrt{-k}\rho}\sinh\sqrt{-k}L \\ 0 & 0 & 1 \end{pmatrix}.$$

4. Up to now, we have neglected the phenomenon of edge focusing, which arises any time the design orbit goes into or emerges from a uniform magnet in non-normal incidence. Starting from the 3×3 transfer matrix for the sector dipole bend magnet in the bending plane,

$$\begin{pmatrix} \cos\Theta & \rho\sin\Theta & \rho\left(1-\cos\Theta\right) \\ -\sin\Theta/\rho & \cos\Theta & \sin\Theta \\ 0 & 0 & 1 \end{pmatrix}$$

derive the 3×3 dispersion transfer matrix for a normal configuration rectangular bend in the bend plane, by assuming that the edge focusing on each side of the rectangular bend may be treated as a thin lens. The total bending angle is denoted by Θ, and you may assume that $\Theta \ll 1$. You may find Wille's discussion helpful.